Online Appendix

1 General Framework

1.1 Non-Strategic Externality

The two rationales we identify in this study are based on the externality of the entry of firms. For complementary markets, the externality is from one market with free entry to other markets. For the model with cost-reducing R&D and spillover, the externality is within the same market. Although it is difficult to generalize the two models to the general case with the nonlinear structure, instead, we could assume away the strategic externality among firms and focus on the externality on social welfare alone. For the model with complementary markets, it is equivalent to assuming that the market equilibrium of the complementary markets has no effect on the firms in industry 0. For the model with R&D and spillover, it is equivalent to assume that the effect of R&D spillover from the entrant has no direct effect on the profit of other firms. The key is that there is positive externality from the entrant on social welfare, in either the same industry or other complementary industries.

There are n firms, which is determined such that the profit of entry is equal to the entry cost; the price function is $p = p(Q)$, and $Q = \sum_{j=1}^{j=n} q_j$, where q_j is the supply of firm j. Social welfare in this market is $sw(Q)$, and there is positive externality, which is measured by $e * Q$. Consequently, the total welfare is measured by $sw(Q) + e * Q$.

Based on the analysis in Mankiw and Whinston (1986), we know that when $e = 0$, there is excessive entry. Obviously, the excessiveness will be lower for a larger externality. Hence, there will be insufficient entry if and only if the externality is sufficiently large. Consequently, we obtain the following result.

Proposition 1 *For free entry with externality, there exists an e^*,*

(1) If $e < e^*$, there is excessive entry.

(2) If $e > e^*$, there is insufficient entry.

The two rationales we identify in this study are both based on the externality of the entry of firms. By assuming away the strategic externality among firms, we focus on the externality on social welfare alone and extend the model to consider nonlinear demand. Due to the trade-off between the business-stealing effect and the externality, there will be insufficient entry if and only if the externality is sufficiently large. Hence, when the externality is sufficiently high, it dominates the business-stealing effect, leading to insufficient entry.

1.2 Complementary Industries

This section considers complementary industries with general demand and cost functions. There are $m+1$ industries, $0, 1, 2 \cdots m$. For industry $1, 2 \cdots m$, there is only one firm in each of these industries, while there is free entry in industry 0 with n firms. Entry cost is f. The inverse demand functions are

$$p_0 = p_0(\sum_{j=1}^{j=n} q_j, Q_j), p_i = p_i(\sum_{j=1}^{j=n} q_j, Q_j),$$

where q_j and Q_j are the supply of firm j in industry 0 and the supply of firm j in industry j, respectively; while p_0 and p_i are prices of product 0 and product i, respectively. The cost of firm j in industry 0 is $c(q_j)$, and the cost of firm j in industry j is $g(Q_j)$.

Given n firms in industry 0 and m firms in the other m industries, we have profits

$$\pi_i = p_i Q_i - g(Q_i), \pi_l = p_0 q_l - c(q_l),$$

where π_i and π_l are profits for firm i in industry i and firm l in industry 0, respectively.

Firm l maximizes π_l by choosing q_l, and firm i maximizes π_i by choosing Q_i. Then we obtain the first-order conditions as follows.

$$\begin{aligned} \frac{\partial \pi_i}{\partial Q_i} &= p_i + \frac{\partial p_i}{\partial Q_i} Q_i - g'(Q_i) = 0, \\ \frac{\partial \pi_l}{\partial q_l} &= p_0 + \frac{\partial p_0}{\partial q_l} q_l - c'(q_l) = 0. \end{aligned}$$

By symmetry, we have $Q_j = Q(n)$ for $j = 1, 2, \cdots m$, and $q_j = q(n)$ for $j = 1, 2, \cdots n$. Then, we obtain the equilibrium quantity. Consequently, the profit for firms in industry 0 is given by $\pi_0 = p_0 q - c(q)$. Free entry requires that the profit of firms in industry 0 is equal to f. Hence, $\pi_0 = p_0 q - c(q) = f$. Let us denote Z as the total output in industry 0, and $Z = Z(n) = n * q(n)$.

For the nonlinear model, we obtain the social welfare $sw(Z(n), q(n), Q(n))$. Social welfare depends on the total production in industry 0 ($Z(n)$), the production of individual firms in industry 0 ($q(n)$), and the production in complementary industries ($Q(n)$). Interested readers can check this relationship for linear demand. Social efficiency requires that

$$sw'(n) = \frac{\partial sw}{\partial Z} Z'(n) + \frac{\partial sw}{\partial q} q'(n) + \frac{\partial sw}{\partial Q} Q'(n) = f.$$

Hence, there is insufficient entry if and only if

$$sw'(n) > \pi_0 \Leftrightarrow \frac{\partial sw}{\partial Z} Z'(n) + \frac{\partial sw}{\partial q} q'(n) + \frac{\partial sw}{\partial Q} Q'(n) > p_0 q - c(q), \text{ where } \frac{\partial sw}{\partial Z}, \frac{\partial sw}{\partial q}, \frac{\partial sw}{\partial Q} > 0.$$

As we know from the literature, $\frac{\partial sw}{\partial Z} Z'(n) + \frac{\partial sw}{\partial q} q'(n) < p_0 q - c(q)$; hence, a necessary condition for insufficient entry is $\frac{\partial sw}{\partial Q} Q'(n) > 0$.

The so-called business-stealing effect is present in the model without complementary markets, which states that the marginal entrant's incentive for entering the market is socially excessive. Therefore, if there is no complementary market so that $\frac{\partial sw}{\partial Q} Q'(n) = 0$, then we have $\frac{\partial sw}{\partial Z} Z'(n) + \frac{\partial sw}{\partial q} q'(n) < \pi_0 = p_0 q - c(q)$; hence, there is excessive entry. Further, if firms are substitutes, meaning that $Q'(n) < 0$, then there is excessive entry as well.

In addition to the business-stealing effect, when there are complementary markets, there is an extra effect of entry. The entry in industry 0 leads to higher output in the other complementary markets, which is welfare improving. Hence, when the number of complementary industries is significantly high, or the size of the complementary market is sufficiently large, this positive effect on welfare could dominate the business-stealing effect. Under both cases, $\frac{\partial sw}{\partial Q} Q'(n)$ is sufficiently large. Consequently, there will be insufficient entry in industry 0.

Further, we would like to make the following remarks.

Remark 1. We fail to obtain the closed-form solution for the general model with nonlinear demand due to the strategic interaction between industries. However, our logic applies to the nonlinear demand case as well.

Remark 2. Based on the analysis in this section, it does not matter whether we assume monopoly firms in the complementary markets, and intuitively, the results remain the same if we consider oligopoly in these markets.

Remark 3. In this section, we assume that the equilibrium always exists, and the price and cost functions satisfy the usual conditions in the literature (e.g. the cost function should be convex.). As we do not rely on these conditions, we omit them in the main body to focus on the key question of concern. They are mainly second-order conditions.[1]

Remark 4. The general case in this section nests the linear case. For the linear case, the demand is linear, which is a special case of the general case. The social welfare for the linear case is

$$sw = anq + amQ - \frac{1}{2}(b(nq)^2 + bmQ^2 + 2rnqmQ + rm(m-1)Q^2) - cnq - cmQ.$$

For the details, please refer to the appendix. The social welfare under the linear case is $sw(Z(n), Q(n))$ due to the linearity. For the general case, social welfare also depends on $q(n)$.

1.3 R&D and Spillover

In this section, we consider cost-reducing R&D and spillover with general demand and cost functions. Formally, the cost of firm i (for $i = 1, 2, \cdots, n$) is $c(q_i, X(x_i, x_j))$, where q_i is the output level of firm i. Further, the cost-reduction due to R&D and spillover from the other firms is equal to $X(x_i, x_j)$, where x_i is the R&D investment of firm i. The cost of R&D is given by $g(x_i)$. We assume that the production cost is decreasing in the total cost-reduction, and the cost-reduction is increasing in firms' R&D. Hence, we have

$$\frac{\partial c(q_i, X(x_i, x_j))}{\partial X} < 0, \frac{\partial X(x_i, x_j)}{\partial x_i} > 0, \frac{\partial X(x_i, x_j)}{\partial x_j} > 0.$$

The inverse market demand is $p = p(\sum_{j=1}^{j=n} q_j)$,where q_j is the output level of firm j.

Firm i's profit is equal to

$$\pi_i = p(\sum_{j=1}^{j=n} q_j)q_i - c(q_i, X(x_i, x_j)) - g(x_i).$$

Firm i maximizes profit by choosing q_i and x_i simultaneously. Then, given n firms, we obtain

[1]

$$\begin{aligned}
\frac{\partial^2 \pi_i}{\partial Q_i^2} &= \frac{\partial p_i}{\partial Q_i} + \frac{\partial p_i}{\partial Q_i} + \frac{\partial^2 p_i}{\partial Q_i^2} Q_i - g''(Q_i) = 2\frac{\partial p_i}{\partial Q_i} + \frac{\partial^2 p_i}{\partial Q_i^2} Q_i - g''(Q_i), \\
\frac{\partial^2 \pi_l}{\partial q_l^2} &= \frac{\partial p_0}{\partial q_l} + \frac{\partial p_0}{\partial q_l} + \frac{\partial^2 p_0}{\partial q_l^2} q_l - c''(q_l) = 2\frac{\partial p_0}{\partial q_l} + \frac{\partial^2 p_0}{\partial q_l^2} q_l - c''(q_l).
\end{aligned}$$

the first-order conditions as follows.

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial q_i} &= p(\sum_{j=1}^{j=n} q_j) + \frac{\partial p(\sum_{j=1}^{j=n} q_j)}{\partial q_i} q_i - \frac{\partial c(q_i, X(x_i, x_j))}{\partial q_i} = 0, \\
\frac{\partial \pi_i}{\partial x_i} &= -\frac{\partial c(q_i, X(x_i, x_j))}{\partial X} \frac{\partial X}{\partial x_i} - g'(x_i) = 0.
\end{aligned}
$$

By symmetry, we have $q_i = q(n), x_i = x(n), X = X(n)$. Let us denote Z as the total output in the market, and $Z = Z(n) = n * q(n)$.

Then the profit of all firms is equal to π.

$$\pi = p(Z(n))q - c(q, X(n)) - g(x).$$

Free entry requires that

$$\pi = p(Z(n))q - c(q, X(n)) - g(x) = f.$$

Social welfare is equal to

$$sw(n) = sw(Z(n), q(n), X(n), x(n)).$$

Social welfare depends on the total production in the industry $(Z(n))$, the production of individual firms $(q(n))$, the total cost-reduction $(X(n))$, and the R&D investment of individual firms $(x(n))$. Interested readers can check this relationship for the linear model.

Social efficiency requires that

$$sw'(n) = \frac{\partial sw}{\partial Z} Z'(n) + \frac{\partial sw}{\partial q} q'(n) + \frac{\partial sw}{\partial X}\frac{\partial X}{\partial n} + \frac{\partial sw}{\partial x} x'(n) = f, \text{ where } \frac{\partial sw}{\partial Z}, \frac{\partial sw}{\partial q}, \frac{\partial sw}{\partial X}, \frac{\partial sw}{\partial x} > 0.$$

Hence, there is insufficient entry if and only if

$$sw'(n) > \pi \Leftrightarrow \frac{\partial sw}{\partial Z} Z'(n) + \frac{\partial sw}{\partial q} q'(n) + \frac{\partial sw}{\partial X}\frac{\partial X}{\partial n} + \frac{\partial sw}{\partial x} x'(n) > p(Z(n))q - c(q, X(n)) - g(x).$$

As we know from the literature, $\frac{\partial sw}{\partial Z} Z'(n) + \frac{\partial sw}{\partial q} q'(n) < \pi$; hence, a necessary condition for insufficient entry is $\frac{\partial sw}{\partial X}\frac{\partial X}{\partial n} + \frac{\partial sw}{\partial x} x'(n) > 0$.

The so-called business-stealing effect is present in the model, so that the marginal entrant's incentive for entry is socially excessive. Therefore, if there is no cost-reducing R&D so that $\frac{\partial X}{\partial n} = x'(n) = 0$, there is excessive entry.

The rationale behind the insufficient entry for higher spillover in the linear model is as follows. A higher spillover rate means that the externality among firms' R&D investment is higher, and the positive effect of an extra firm on R&D spillover is stronger. Consequently, the total cost-reduction X increases in the number of firms. In other words, $X(n)$ or $X'(n)$ is sufficiently high. The entrants do not take this positive R&D spillover effect on cost-reduction into account. Hence, when the spillover rate is sufficiently high, the positive effect on cost-reduction dominates the business-stealing effect, leading to insufficient entry.

Further, we would like to make the following remarks.

Remark 1. We fail to obtain the closed-form solution for the general model due to the strategic interaction between firms. However, our intuition applies for the general model as well.

Remark 2. In this section, we assume that the equilibrium always exists, and the price and cost functions satisfy the usual conditions in the literature. We omit them in the main body to focus on the key question of concern. They are mainly second-order conditions.[2]

---

[2]

$$\begin{aligned}
\frac{\partial^2 \pi_i}{\partial q_i^2} &= 2\frac{\partial p}{\partial q_i} + \frac{\partial^2 p}{\partial q_i^2} Q_i - \frac{\partial^2 c(q_i, X(x_i, x_j))}{\partial q_i^2}, \\
\frac{\partial^2 \pi_i}{\partial x_i^2} &= -\frac{\partial^2 c(q_i, X(x_i, x_j))}{\partial X^2} (\frac{\partial X}{\partial x_i})^2 - \frac{\partial c(q_i, X(x_i, x_j))}{\partial X} \frac{\partial^2 X}{\partial x_i^2} - g''(x_i), \\
\frac{\partial^2 \pi_i}{\partial q_i \partial x_i} &= -\frac{\partial^2 c(q_i, X(x_i, x_j))}{\partial X \partial q_i} \frac{\partial X}{\partial x_i}.
\end{aligned}$$

The second-order condition requires that

$$\frac{\partial^2 \pi_i}{\partial q_i^2} < 0, \frac{\partial^2 \pi_i}{\partial x_i^2} < 0, \frac{\partial^2 \pi_i}{\partial q_i^2} * \frac{\partial^2 \pi_i}{\partial x_i^2} - (\frac{\partial^2 \pi_i}{\partial q_i \partial x_i})^2 > 0.$$